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A hybrid reconstruction algorithm for electrical impedance tomography

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Abstract

Electrical impedance tomography (EIT) is a technique for reconstructing the conductivity distribution inside an inhomogeneous distribution by injecting currents at the boundary of a subject and measuring the resulting changes in voltage. A hybrid method is proposed for solving the inverse problem for EIT, which combines the Krylov subspace and the Tikhonov regularization for double levels of regularization to the ill-posed problem. Numerical simulation results using the hybrid method are presented and compared to those from truncated singular value decomposition (TSVD) regularization and the Tikhonov regularization. Experimental results with the hybrid method are also presented, indicating that the hybrid method can reduce the computation time, and improve the resolution of reconstructed images with the regularization parameter automatically chosen by the L-curve method.

Keywords: electrical impedance tomography, image reconstruction, Krylov subspace, hybrid method

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Electrical impedance tomography (EIT) has been widely used in three principal areas: medical imaging, industrial process imaging and geophysical surveying (Webster 1990). The physical model for EIT can be written as

$$\mathbf{V} = \mathbf{U}(\boldsymbol{\sigma}; \mathbf{I}) = \mathbf{R}(\boldsymbol{\sigma})\mathbf{I} \quad (1)$$

where $\mathbf{V}$ is the measured voltage vector on the electrodes surrounding the periphery of a subject, $\mathbf{I}$ is the injected current vector, $\boldsymbol{\sigma}$ is the conductivity distribution in a cross section of the subject, $\mathbf{U}(\boldsymbol{\sigma}; \mathbf{I})$ is the forward model mapping $\boldsymbol{\sigma}$ and $\mathbf{I}$ to $\mathbf{V}$ and $\mathbf{R}(\boldsymbol{\sigma})$ is the model mapping $\boldsymbol{\sigma}$ to resistance.

This model depends nonlinearly on the conductivity $\boldsymbol{\sigma}$ and linearly on the current $\mathbf{I}$ (Vauhkonen 1997). The aim of image reconstruction for EIT is to obtain the conductivity distribution $\boldsymbol{\sigma}$ using the boundary voltage vector $\mathbf{V}$ and injected current vector $\mathbf{I}$.

Because the inverse problem is both nonlinear and ill-posed, little noise in the measured data can cause large errors in the estimated conductivity (Borsic 2002, Vauhkonen *et al* 1998). Because the measurements depend nonlinearly on the conductivity, some nonlinear algorithm, such as the regularized Gauss–Newton method, should be used, which requires repeated and intensive computation. Therefore, this is not feasible in a real time system. In principle, a small change in conductivity can be reconstructed accurately by considering the linearized problem (Vauhkonen 1997). A Jacobian matrix is a linear approximation at $\boldsymbol{\sigma}_0$ of the nonlinear model. It describes the changes in the measured voltages on the electrodes due to small changes in conductivity of the elements in a cross section.

In order to obtain a stable solution, regularization techniques are adopted. Two main types of regularization method have been widely used to solve the inverse problem. One is categorized as the penalty methods, such as the Tikhonov regularization, in which a penalty term is added

to filter out unwanted components. The other is categorized as the projection methods, such as the truncated singular value decomposition (TSVD) regularization, which means that the solution is projected into a specific subspace without the unwanted components (Jacobsen 2004).

In the hybrid methods, a subspace (e.g. Krylov subspace) method is combined with a regularization method to solve the projected problem (Jacobsen 2004). With the penalty methods all components in an equation are regularized, which are contaminated by random noise and cannot be used if the boundary conditions are unknown. Therefore, these methods need lengthy calculations. Although the projection methods can have a regularizing effect by limiting the number of subspace dimensions, it is difficult to determine the optimal number of subspace dimensions, and therefore the unwanted components may be retained or the wanted components may be truncated. The so-called hybrid methods attempt to solve these problems by adding a regularization operator to the relatively small inner least-squares problems (Hanke and Hansen 1993). For example, a hybrid algorithm originating from the papers by O'Leary and Simmons (1981) and Bjorck (1988) is fast and accurate.

2. Typical reconstruction algorithms

2.1. Linear approximation

Under some assumptions the inverse problem for EIT can be solved by linear approximation accurately and rapidly. Let us consider some associated linearization procedures. At first, by ignoring higher order terms, a linear approximation of an EIT model takes the following form:

$$\delta \mathbf{U} = \mathbf{U}'(\sigma_0) \delta \sigma = J \delta \sigma \quad (2)$$

where $\delta \sigma$ is the change in conductivity, $\delta \mathbf{U}$ is the perturbation of boundary voltage due to the change of σ and J is the Jacobian matrix at σ_0 , i.e., the partial derivatives of voltages with respect to conductivity.

For fast calculation of the Jacobian matrix, a sensitivity method based on Geselowitz's sensitivity theorem (Polydorides and Lionheart 2002) can be used:

$$J_{dmk} = \frac{\delta \mathbf{U}_{dm}}{\delta \sigma_k} = - \int_{\Omega_k} \nabla u(\mathbf{I}^d) \cdot \nabla u(\mathbf{I}^m) dV \quad (3)$$

where $u(\mathbf{I}^d)$ is the potential for the d th drive pattern and $u(\mathbf{I}^m)$ is the hypothetical measurement potential (Holder 2004).

Image reconstruction for EIT is a typical ill-posed inverse problem, in which a solution either does not exist or is not unique or unstable. In practice, a generalized solution can be obtained. By applying regularization techniques, a modified operator is introduced to guarantee that the inverse problem is bounded and a stable least-squares solution can be obtained. There are two types of regularization methods (Hansen 2005): the penalty methods and the projection methods. Using a penalty method the ill-posed problem can be converted to a well-posed problem by adding a penalty term, while with a projection method a rank-deficient problem is created. Though they work in different ways, nearly identical results may be obtained by the two types of regularization methods.

2.2. Projection methods

TSVD regularization is a typical projection method, which directly converts the ill-posed problem into a rank-deficient approximation. The SVD of a Jacobian matrix takes the form

$$J = \tilde{U} \Sigma \tilde{V}^T \quad (4)$$

where $\tilde{U} = [\tilde{u}_1 \cdots \tilde{u}_m]$, $\Sigma = \text{diag}(s_1 \cdots s_r)$ and $\tilde{V} = [\tilde{v}_1 \cdots \tilde{v}_n]$.

The TSVD solution is given by

$$\delta \sigma^{(k)} = \sum_{i=1}^k \frac{\tilde{u}_i^T \delta \mathbf{U}}{s_i} \tilde{v}_i. \quad (5)$$

The first k singular vectors form the best k -dimensional subspace, which is used to project the solution. Because the SVD computation which is of order $O(\min(m, n)mn)$ FLOPS is complicated (Hansen 2005), it is difficult to use for real-time image reconstruction.

2.3. Tikhonov regularization

Tikhonov regularization is a penalty method, which converts the ill-posed problem to a well-posed problem by adding a penalty term. For solving the inverse problem, the penalty term can be written in the form (Vauhkonen 1997)

$$\min_{\sigma} \{ \|\mathbf{V} - \mathbf{U}(\sigma)\|^2 + \lambda^2 \|L\sigma\|^2 \} \quad (6)$$

where σ is the conductivity distribution, $\mathbf{U}(\sigma)$ are the calculated potentials obtained from the model with known σ , $\mathbf{V}$ are the measured potentials on the electrodes, L is a regularization matrix and λ is a regularization parameter.

The most often used regularization matrices in EIT are the identity matrix, and the matrices corresponding to the first-order and second-order difference operators. The former is to minimize the norm of the solution only and the latter can lead to a continuous solution. The choice of regularization parameter is very important because an inappropriate parameter can lead to a useless result. There are three methods for choosing the regularization parameter (Vauhkonen 1997) when the magnitude of noise is unknown. They are the quasi-optimality criterion, the generalized cross-validation (GCV) and the L-curve criterion. The generalized cross-validation (GCV) is based on the statistical estimation theory and the L-curve method is based on heuristic observations. The L-curve attempts to balance the penalty term $\|L\sigma\|$ versus the residual norm $\|\mathbf{V} - \mathbf{U}(\sigma)\|$.

3. Hybrid method

3.1. Krylov subspace

With a projection algorithm a solution is found effectively lying in a low-dimensional specific subspace. The first k singular vectors are the 'optimal' set of basis vectors for a particular matrix J . Since SVD is time-consuming for large-scale problems, the singular vectors cannot be calculated in real time. Not only can the Krylov subspace be quickly calculated, but also it is suitable to the particular problem because the

Krylov subspace is closely associated with the Jacobian J and $\delta\mathbf{U}$.

Definition 1 (Krylov subspace). The Krylov subspace of dimension k generated by the matrix J and vector $\delta\mathbf{U}$ is defined as (Jacobsen 2004)

$$K_k(J^T J, J^T \delta\mathbf{U}) \equiv \text{span}\{J^T \delta\mathbf{U}, (J^T J)J^T \delta\mathbf{U}, (J^T J)^2 J^T \delta\mathbf{U}, \dots, (J^T J)^{k-1} J^T \delta\mathbf{U}\}. \quad (7)$$

The subspace K_k carries important information about the problem. By ortho-normalizing the vectors of K_k , ortho-normal unit basis vectors compose the subspace W_k , which is well suited to play a role in the projection method for discrete ill-posed problems.

In order to compute the basis vectors w_i of W_k efficiently, the Lanczos bidiagonalization algorithm (Hansen 2005) is used. The algorithm applies the matrix J and the starting vector $\delta\mathbf{U}$ to create the Krylov subspace. The algorithm creates a lower bi-diagonal matrix $B_k \in R^{k+1 \times k}$ and two orthogonal matrices $\bar{V}_k$ and $\bar{U}_{k+1}$, and their relationships are given by

$$\bar{U}_{k+1}(\beta_0 e_1) = \delta\mathbf{U} \quad (8)$$

$$J \bar{V}_k = \bar{U}_{k+1} B_k \quad (9)$$

$$J^T \bar{U}_{k+1} = \bar{V}_k B_k^T + \alpha_{k+1} \bar{v}_{k+1} e_{k+1}^T. \quad (10)$$

The lower bi-diagonal matrix $B_k \in R^{k+1 \times k}$ takes the following form:

$$B_k = \begin{bmatrix} \alpha_1 & & & & \\ \beta_1 & \alpha_2 & & & \\ & \beta_2 & \ddots & & \\ & & \ddots & \alpha_k & \\ & & & \beta_k & \end{bmatrix} \quad (11)$$

where $e_i = (0, \dots, 0, 1_i, 0, \dots, 0)^T \in R^{k+1}$ means that the i th element of e is 1 while others are 0, the scalars $\alpha_i \geq 0$ and $\beta_i \geq 0$ are chosen so that $\|\bar{u}_i\| = \|\bar{v}_i\| = 1$, and they are solved using the following equation:

$$\begin{cases} \beta_0 \bar{u}_1 = \delta\mathbf{U}, & \alpha_1 \bar{v}_1 = J^T \bar{u}_1 \\ \beta_i \bar{u}_{i+1} = J \bar{v}_i - \alpha_i \bar{u}_i & i = 1, 2, \dots \\ \alpha_{i+1} \bar{v}_{i+1} = J^T \bar{u}_{i+1} - \beta_i \bar{v}_i \end{cases} \quad (12)$$

The matrix $\bar{V}_k = (\bar{v}_1, \dots, \bar{v}_k)$ forms an orthogonal basis of the Krylov subspace $K_k(J^T J, J^T \delta\mathbf{U})$, while $\bar{U}_{k+1} = (\bar{u}_1, \dots, \bar{u}_{k+1})$ forms an orthogonal basis of the Krylov subspace $K_{k+1}(J J^T, \delta\mathbf{U})$.

3.2. Projection method based on Krylov subspace

As explained above, the Krylov subspace is a subspace (Jacobsen 2004) suited to the particular problem. It is spanned from the columns of the matrix $W_k = (w_1, \dots, w_k) \in R^{n \times k}$, and a solution is obtained in this basis that fits the right-hand side $\delta\sigma$. The constrained least-squares problem can be formulated as

$$\min_x \|J\delta\sigma - \delta\mathbf{U}\|_2 \quad \text{s.t.} \quad x \in W_k = \text{span}\{w_1, \dots, w_k\}. \quad (13)$$

Also the constraint can be expressed as the requirement that $\delta\sigma = W_k y$, where $y \in R^k$ is the new unknown. This leads to a regularized solution.

$$\delta\sigma^{(k)} = W_k y_k, \quad y^k = \arg \min_y \|(J W_k) y - \delta\mathbf{U}\|_2. \quad (14)$$

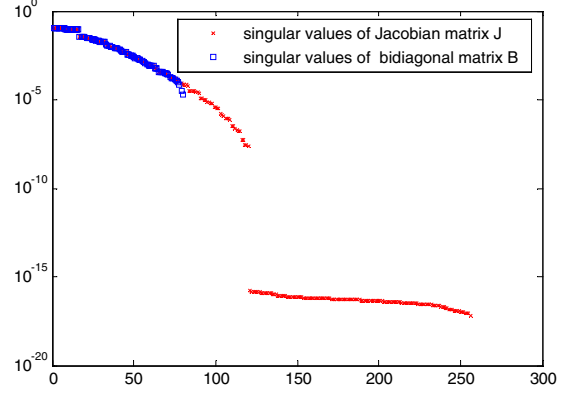


Figure 1. Singular value of the Jacobian and bi-diagonal matrix on a logarithmic scale.

The least-squares problem $\|(J W_k) y - \delta\mathbf{U}\|_2$ is a projected problem, because it is obtained by projecting the original problem onto the subspace $\text{span}\{w_1, \dots, w_k\}$.

In fact, the ortho-normal basis vectors $w_1, \dots, w_k$ and the singular vectors have the same feature that each vector w_i is mainly dominated by the singular vectors $\bar{v}_i$. Similarly, the singular values of the Jacobian and the projected Jacobian matrix are almost identical for the left k th singular values as shown in figure 1. Furthermore, for the isotropic conductivity case, the condition numbers of the original problem and the projected problem are 5.215×10^6 and 6.277×10^3 respectively, when the Krylov subspace dimension is selected as 80 (Note that the 80th singular value is almost equal to the typical noise level in real measurement systems with 16 electrodes). Therefore, the Krylov subspace not only reduces the condition number prodigiously, enhances the noise immunity, and preserves the useful information of the original problem, but also reduces the computational load. However, it is difficult to choose the dimension of the Krylov subspace optimally.

3.3. Proposed hybrid method

A hybrid method combines a subspace method with a regularization method for solving the projected problem (Hansen 2005). Some regularization methods can be applied to enforce a second level of regularization to the Krylov subspace. The projection method using the Krylov subspace takes the form of equation (14). Combining the Krylov method with the Tikhonov regularization leads to

$$\begin{aligned} \delta\sigma_\lambda^{(k)} &= W_k y_\lambda^{(k)}, \\ y_\lambda^{(k)} &= \arg \min_y \{ \|(J W_k) y - \delta\mathbf{U}\|_2^2 + \lambda^2 \|R y\|_2^2 \} \end{aligned} \quad (15)$$

where $R = L \times W_k$ is a regularization matrix of the Krylov subspace.

Using equations (8) and (9), the least-squares residual can be reformulated as

$$\begin{aligned} \|(J W_k) y - \delta\mathbf{U}\|_2 &= \|\bar{U}_{k+1} B_k y - \delta\mathbf{U}\|_2 \\ &= \|\bar{U}_{k+1}^T (\bar{U}_{k+1} B_k y - \delta\mathbf{U})\|_2 \\ &= \|B_k y - \bar{U}_{k+1}^T \delta\mathbf{U}\|_2 \\ &= \|B_k y - \beta_0 e_1\|_2 \end{aligned} \quad (16)$$

where $e_1 = (1, 0, \dots, 0)^T \in R^{k+1}$

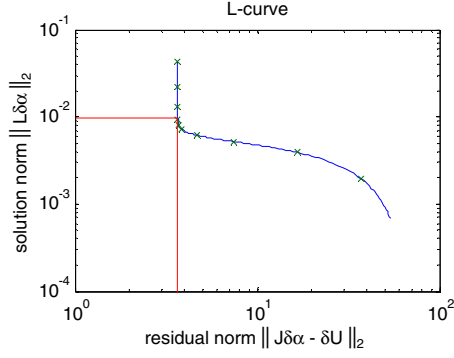


Figure 2. L-curve for the Tikhonov regularization: a log–log plot of the solution norm $\|L\delta\sigma\|_2$ versus residual norm $\|J\delta\sigma - \delta U\|_2$ with regularization parameter λ chosen at the corner of the curve.

Table 1. Comparison of three different algorithms in terms of computation time (in seconds). The dimension of subspace is 82 and the regularization parameters of the other two methods are automatically chosen using the L-curve method.

Number of elements	Imaging reconstruction algorithms (second)		
	TSVD	Tikhonov regularization	Hybrid method
534	0.2845	0.1327	0.0224
1234	0.4152	0.6812	0.0268
2136	0.6883	2.4054	0.0332
4936	1.4522	9.1288	0.0517

Consequently, the alternative form of equation (15) can be

$$\begin{aligned} \delta\sigma_{\lambda}^{(k)} &= W_k y_{\lambda}^{(k)}, \\ y_{\lambda}^{(k)} &= \arg \min_y \{ \|B_k y - \beta_0 e_1\|_2^2 + \lambda^2 \|Ry\|_2^2 \}. \end{aligned} \quad (17)$$

Due to the bi-diagonal structure of B_k , the projected Tikhonov problem expressed by equation (17) can be solved in $O(k)$ operations. In contrast, TSVD and the Tikhonov regularization are the same in complexity, i.e. $O(\min(m, n)mn)$, because TSVD requires computation of SVD and the Tikhonov regularization requires a solution to the least-squares problem (or SVD). In table 1 three different algorithms are compared in terms of computation time with an Intel Pentium 4 PC, 2.93 GHz CPU with 512 MB RAM running MATLAB® version 7.0.4 on a Microsoft Windows XP® platform. In addition,

$$\|L\delta\sigma_{\lambda}^{(k)}\|_2 = \|RW_k^T W_k y_{\lambda}^{(k)}\|_2 = \|Ry_{\lambda}^{(k)}\|_2 \quad (18)$$

$$\|J\delta\sigma_{\lambda}^{(k)} - \delta U\|_2 = \|B_k y - \beta_0 e_1\|_2. \quad (19)$$

It is concluded that the solution norm $\|L\delta\sigma_{\lambda}^{(k)}\|_2$ and the residual norm $\|J\delta\sigma_{\lambda}^{(k)} - \delta U\|_2$ can be obtained directly from the Tikhonov solution $y_{\lambda}^{(k)}$ for the projected problem, since the columns of W_k are orthogonal basis vectors for the Krylov subspace K_k . Having obtained the residual norm and the solution norm, the regularization parameter can be chosen directly. The L-curve is shown in figure 2. The optimal projected solution $y_{\lambda}^{(k)}$ can be solved by multiplication with W_k , and the optimal solution to the original ill-posed problem can be obtained.

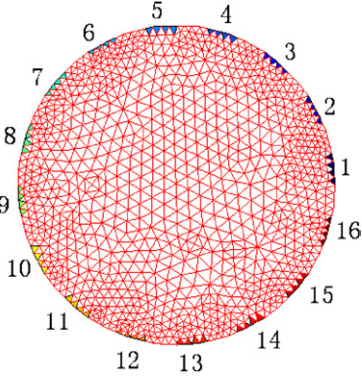


Figure 3. Finite element mesh used for calculating voltages. The locations of electrodes are marked with coloured elements and the corresponding electrode numbers are labelled.

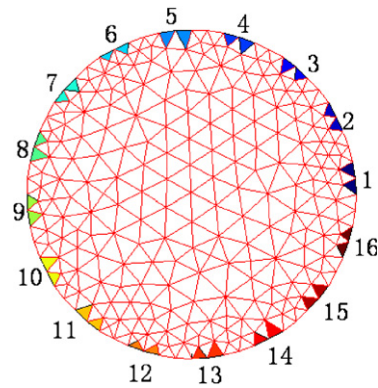


Figure 4. Finite element mesh used for image reconstruction. The locations of electrodes are marked with coloured elements and the corresponding electrode numbers are labelled.

4. Experimental results

4.1. Numerical simulation

In EIT, the forward problem can be solved using a complete electrode model and a finite element method. A mesh of 1856 first-order triangular elements, 993 nodes and 16 electrodes are used for the forward calculations (see figure 3). An adjacent current injection pattern is used. The reconstructed image presents static conductivity values using the mesh with 464 first-order triangular elements, 265 nodes and 16 electrodes (see figure 4). To solve the forward problem, a mesh is obtained by dividing one element used in the inverse calculation into four elements.

Two conductivity distributions, as shown in the first row of figure 5, are simulated to verify the new algorithm. The first simulated distribution contains three objects, and the second one contains six objects. $\pm 0.1\%$ Gauss random noise, corresponding to the typical noise level in a real measurement system, was added to the simulated voltages.

For the TSVD method the subspace dimension k is chosen in the best situation and the regularization parameters are chosen by the L-curve method for the other methods. As shown in figure 5, the TSVD method cannot achieve such a good result as the Tikhonov regularization and the hybrid method. However, it is difficult to tell the effect of the

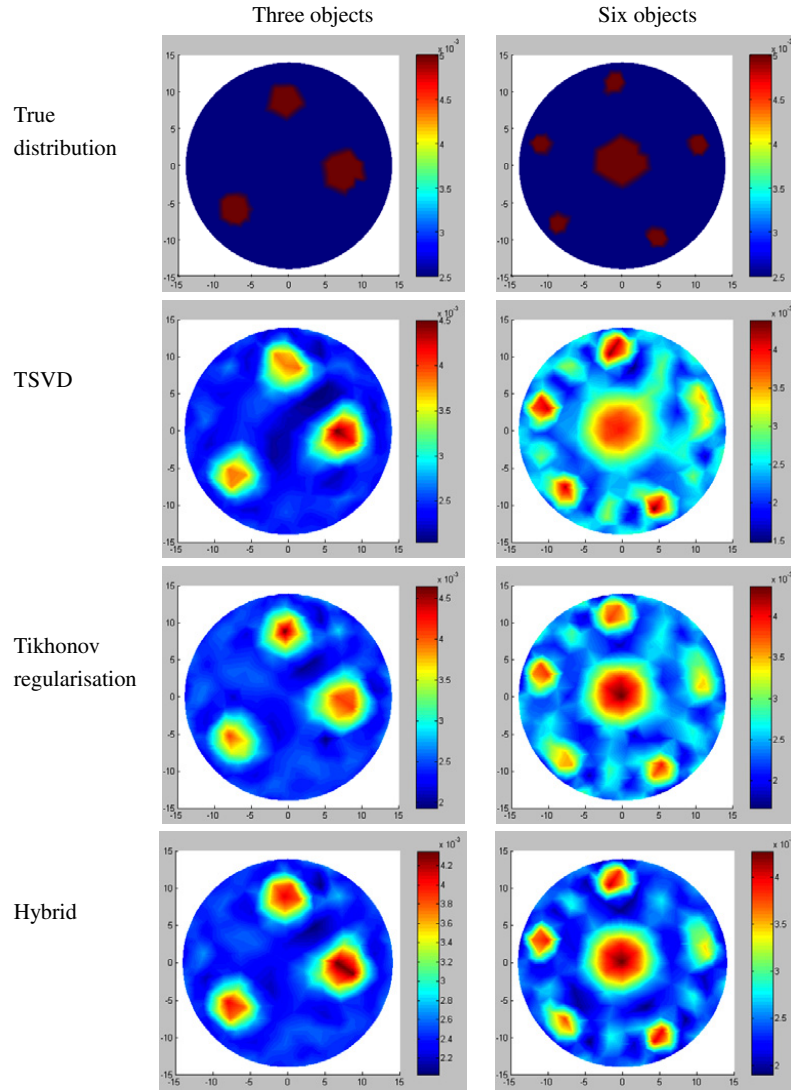


Figure 5. Image reconstruction with different algorithms.

Tikhonov regularization from the hybrid method directly. Therefore, it is necessary to employ some evaluation indicator to evaluate the performance of the three image reconstruction algorithms. Since for simulation the real conductivity distribution can be obtained easily, the relative image error is used and it is defined as $\frac{\|\hat{\sigma} - \sigma\|}{\|\sigma\|}$, where $\hat{\sigma}$ is the calculated conductivity and σ is the real one.

The simulation results show that the hybrid method is the best one among the three methods.

4.2. Experiments

An EIT system was used with a single current source, adjacent current (500 kHz, 2 mA sinusoidal ac) injection and adjacent voltage detection. The diameter of the tank is 125 mm and the size of each electrode is 19×77 mm. The true distributions and reconstructed images are shown in figure 6.

It can be seen from figure 6 that the hybrid method can produce good images, in which the size and position of the rods are correctly reconstructed. The regularization matrix is the first-order difference operator, and the regularization

parameter is automatically chosen by the L-curve method and the dimension of the Krylov subspace is 80, which is justified to the EIT system with 16 electrodes. The computation time for the reconstruction of two plastic rods as shown in figure 6(a) and two big plastic rods and two small ones as shown in figure 6(c) is 0.0230 s and 0.0231s, respectively.

5. Conclusion and future work

The presented hybrid method is a combination of the Krylov subspace with the Tikhonov regularization for solving the projected problem. As the singular vectors, the first k basis vectors in the Krylov subspace contain the optimal information of the Jacobian matrix and can be calculated quickly. According to table 2, the hybrid method can improve the quality of reconstructed images, and the accuracy of the recovered conductivity values. According to table 1, the computation time of the hybrid method is much less than the TSVD and the Tikhonov regularization when the subspace is appropriately chosen. Therefore, it is suitable for on-line operation, such as for multi-phase flow measurement.

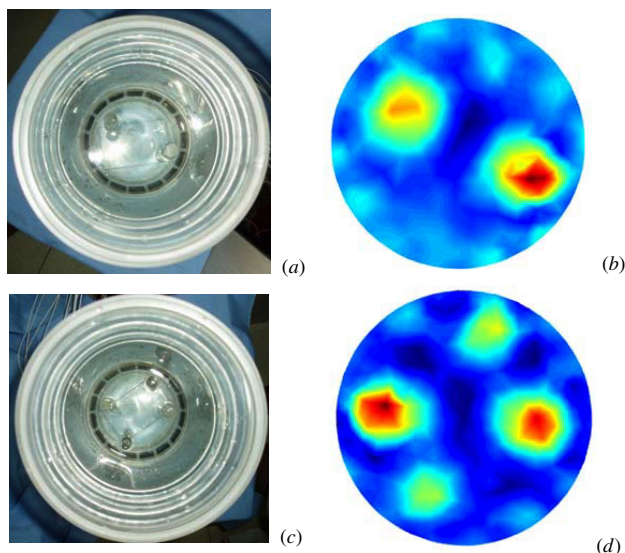


Figure 6. Experimental results. (a) Two plastic rods were immersed in saline water and data were collected from 16 electrodes; (b) reconstruction of (a) using experimental data and hybrid method; (c) two big plastic rods and two small ones were immersed in saline water and data were collected with 16 electrodes; (d) the reconstruction of (c) using experimental data and the hybrid method.

Table 2. Comparison of three different algorithms in terms of relative image error.

Number of objects	Imaging reconstruction algorithms		
	TSVD	Tikhonov regularization	Hybrid method
3	0.132 38	0.132 85	0.127 02
6	0.185 86	0.178 44	0.171 77

With the regularization parameter automatically chosen by the L-curve method, good results have been obtained. Furthermore, a number of regularization algorithms can be split into independent modules and different projection methods can have a natural modular formulation. The hybrid methods can construct the modules and combine them (Jacobsen 2004). Some specific combinations may

be used to solve a particular problem. Other combinations are currently under investigation to further improve the quality of reconstructed images for EIT.

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